EMF. Induced field.
Displacement current.

ACT: Two rings

Two copper rings with the same geometry move toward identical magnets with the same velocity as shown. The ring in case 2, though, has a small slit. Compare the magnitudes of:

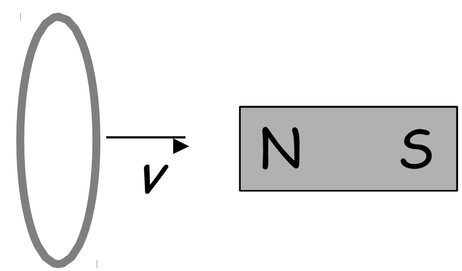
- the induced emf's in the rings:

  A. $\varepsilon_1 < \varepsilon_2$

  B. $\varepsilon_1 = \varepsilon_2$

  C. $\varepsilon_1 > \varepsilon_2$

EMF is independent of the actual ring. It would be the same for a wooden ring, or even in vacuum!



1

- the magnetic force on the rings:

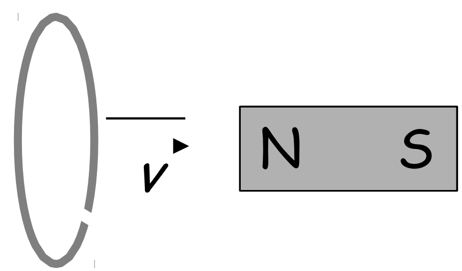
  A. $F_1 < F_2$

  B. $F_1 = F_2$

  C. $F_1 > F_2$

No current in case 2 because ring is open.

$F_2 = 0$



2

In-class example: AC generator

A coil of copper wire consists of 120 loops of wire wound around a 10 cm × 20 cm rectangular form. If this coil is used to generate an induced emf by rotating it in a 0.3 T field at 60 Hz, what is the maximum emf that can be produced?

A. 33 V

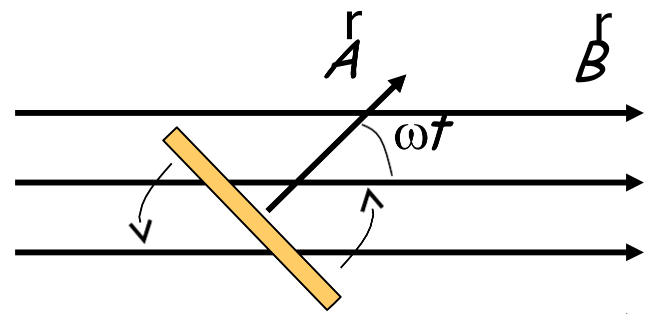
B. 43 V

C. 120 V

D. 217 V

E. 27 100 V

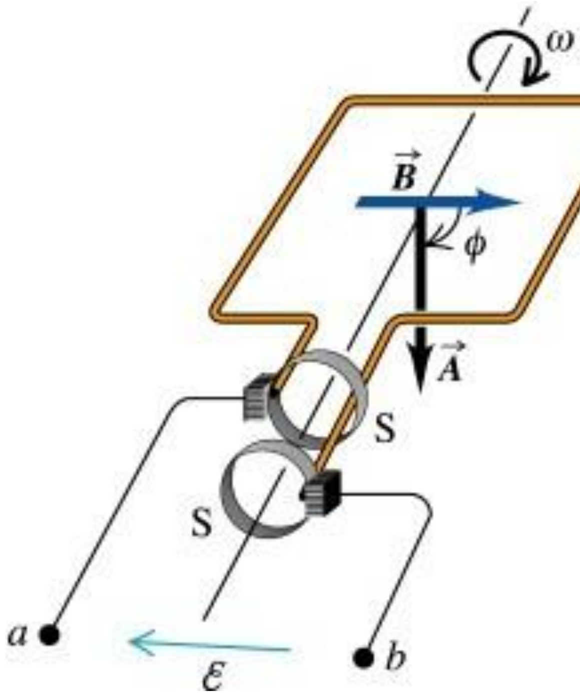
$$\Phi_B = N\vec{B} \times \vec{A}$$
$$= NBA\cos(\omega t)$$

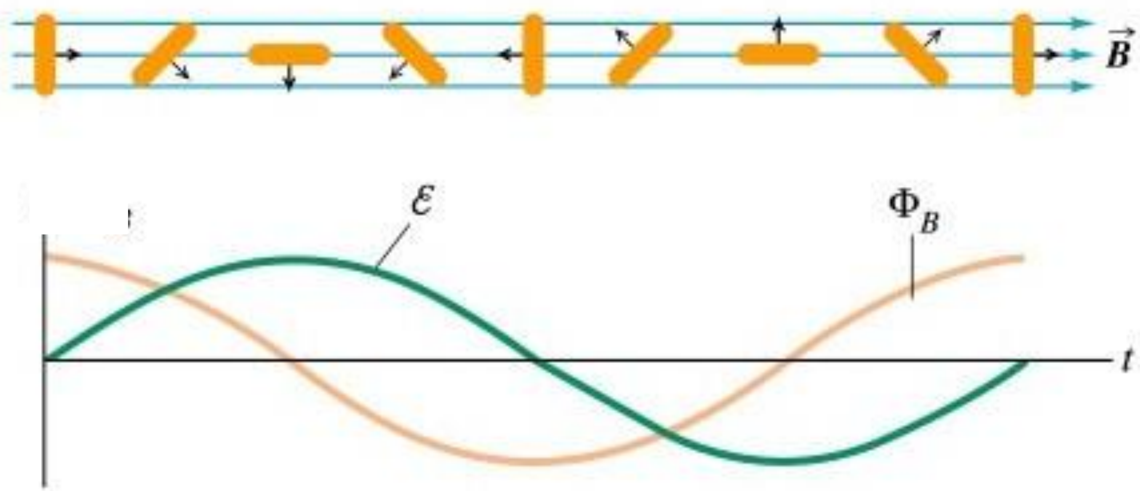


$$\varepsilon = -\frac{d\Phi_B}{dt} = \omega NBA\sin(\omega t)$$

$$\varepsilon_{max} = \omega NBA$$
$$= \left(60\,\frac{\text{turns}}{s}\,\frac{2\pi\text{ rad}}{1\text{ turn}}\right)(120)(0.3\text{ T})(0.1\times 0.2\text{ m}^2)$$
$$= 271\text{ V}$$

This is an AC generator!





$$\Phi_B = BA\cos(\omega t)$$

$$\varepsilon = NBA\omega \sin(\omega t)$$

AC generator

Water turns wheel

- ◊ rotates magnet
- ◊ changes flux
- ◊ induces emf
- ◊ drives current

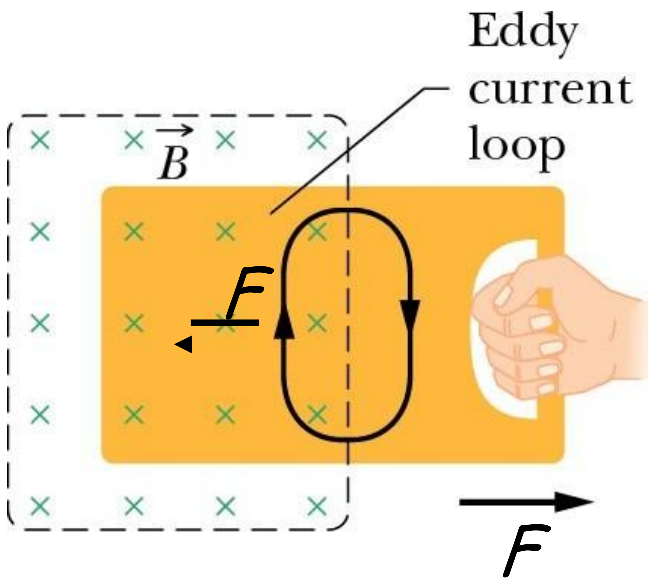
DEMOs:

Loop rotating between electromagnets

Hand-driven generator with bulbs

Eddy currents

Pull a sheet of metal through B-field



Induced emf due to change in flux
$\rightarrow$ currents still generated but
path they take not well-defined
$\rightarrow$ eddy currents

External force required to pull sheet because induced current opposes change.
Current through bulk $\rightarrow$ heating $\rightarrow$ energy loss

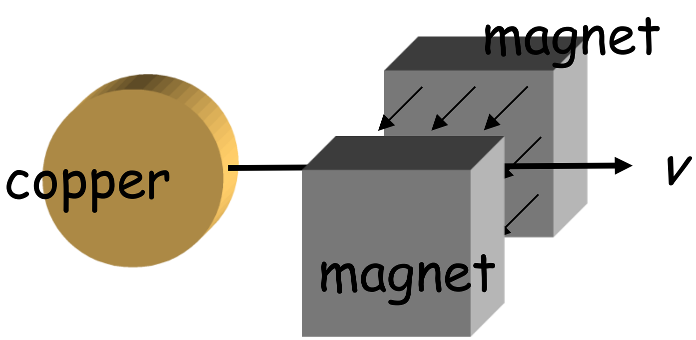
Applications: Damping

Eddy currents induced in a copper plate when it passes near magnets.

Low resistivity ↔ large currents

→ Magnets exert force against motion

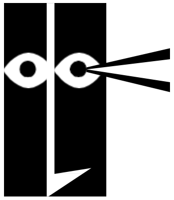
→ Plate is slowed down



This effect does not "wear down" (like rubbing surfaces do). It is used as a brake system in roller coasters and alike.

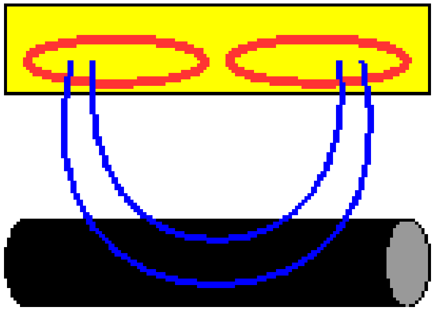
To reduce eddy currents when undesirable, prevent currents from flowing (cutting slots or laminating material).

DEMOs:
Eddy currents:
pendulum and magnets
through tube



Metal detectors

- Pulse current through primary coil
  - B-field which changes with time
  - induces currents in a piece of nearby metal
- Induced currents generate B-field which changes in time
  - induces currents in coils of metal detector
  - sets off signal, alarm…

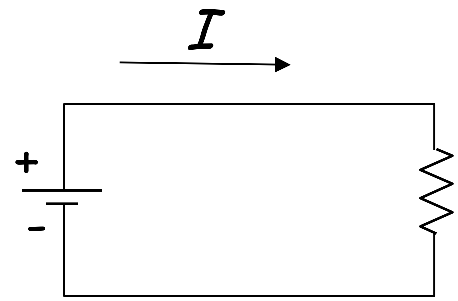


Back to the EMF

EMF of a battery (from Phys 221):

$$\varepsilon = V_+ - V_- = -\int_-^+ \vec{E} \cdot d\vec{l} = \int_+^- \vec{E} \cdot d\vec{l}$$

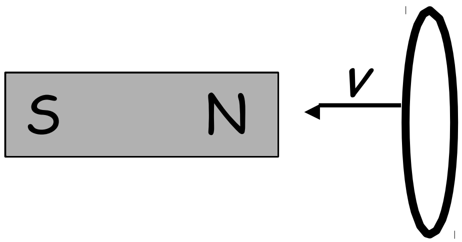
Integrating in the direction of the current



$$\varepsilon = \int \vec{E} \cdot d\vec{l}$$

Motional emf

A loop moves towards a magnet
$\Rightarrow$ a current is induced.



Cause: Magnetic force on the moving charges in the loop: $\vec{F} = q\vec{v} \times \vec{B}$

Work: $W = \int \vec{F} \cdot d\vec{l} = \int q\left(\vec{v} \times \vec{B}\right) \cdot d\vec{l}$

If this was an electric force, the corresponding emf would be

$$\varepsilon = \frac{W}{q} = \frac{\int \vec{F} \cdot d\vec{l}}{q} = \int \left(\vec{v} \times \vec{B}\right) \cdot d\vec{l}$$

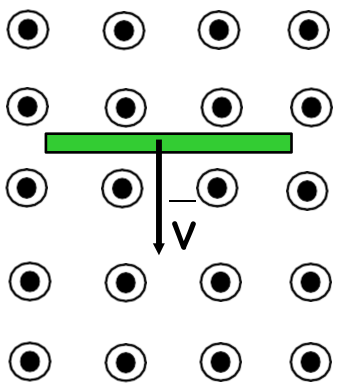
Motional emf $\quad \varepsilon = \int \left(\vec{v} \times \vec{B}\right) \cdot d\vec{l}$

Note: This does not come from an electric field.

ACT: E-field in an open circuit

What is the direction of the E-field in the moving conductor?

A. Left
B. Right
C. Up
D. Down



$\bar{B}$ constant with time

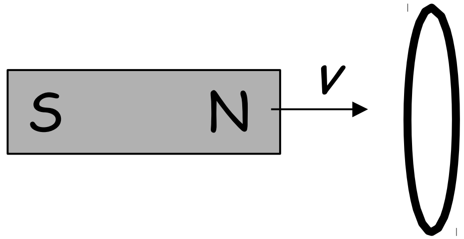
Force points left.

Positive charge accumulates at left end.

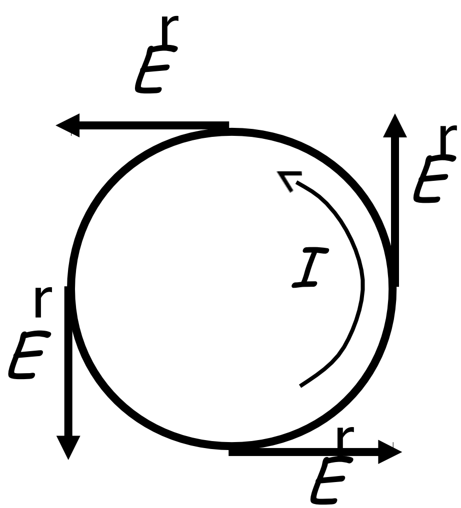
Electric field points right. But this a regular electrostatic E field produced by charges.

Induced electric field

A magnet moves towards a loop $\Rightarrow$ a current is induced.



No magnetic forces involved. $\Rightarrow$ There must be an (induced) electric field!



$$\boxed{\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}}$$

Changing *B*-field induces an *E*-field.
No charges involved!!

Faraday's law links *E* and *B* fields

q $\longrightarrow$ generates *E*-field $\longrightarrow$ q experiences force

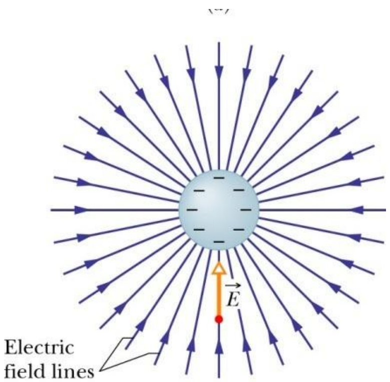
changing *B*-field
generates an *E*-field

moving q $\longrightarrow$ generates *B*-field $\longrightarrow$ moving q experiences force

Two types of E fields

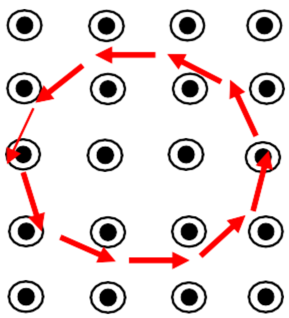
1. E field produced by charges
   - Lines begin/end on charges
   - $\oint \vec{E} \cdot d\vec{l} = 0$ (on a closed loop)

   $\Rightarrow$ Conservative electric field



2. E field produced by B field that changes with time
   - lines are loops that do not begin/end
   - $\oint \vec{E} \cdot d\vec{l} = \varepsilon \; (\neq 0)$

   $\Rightarrow$ Non conservative electric field

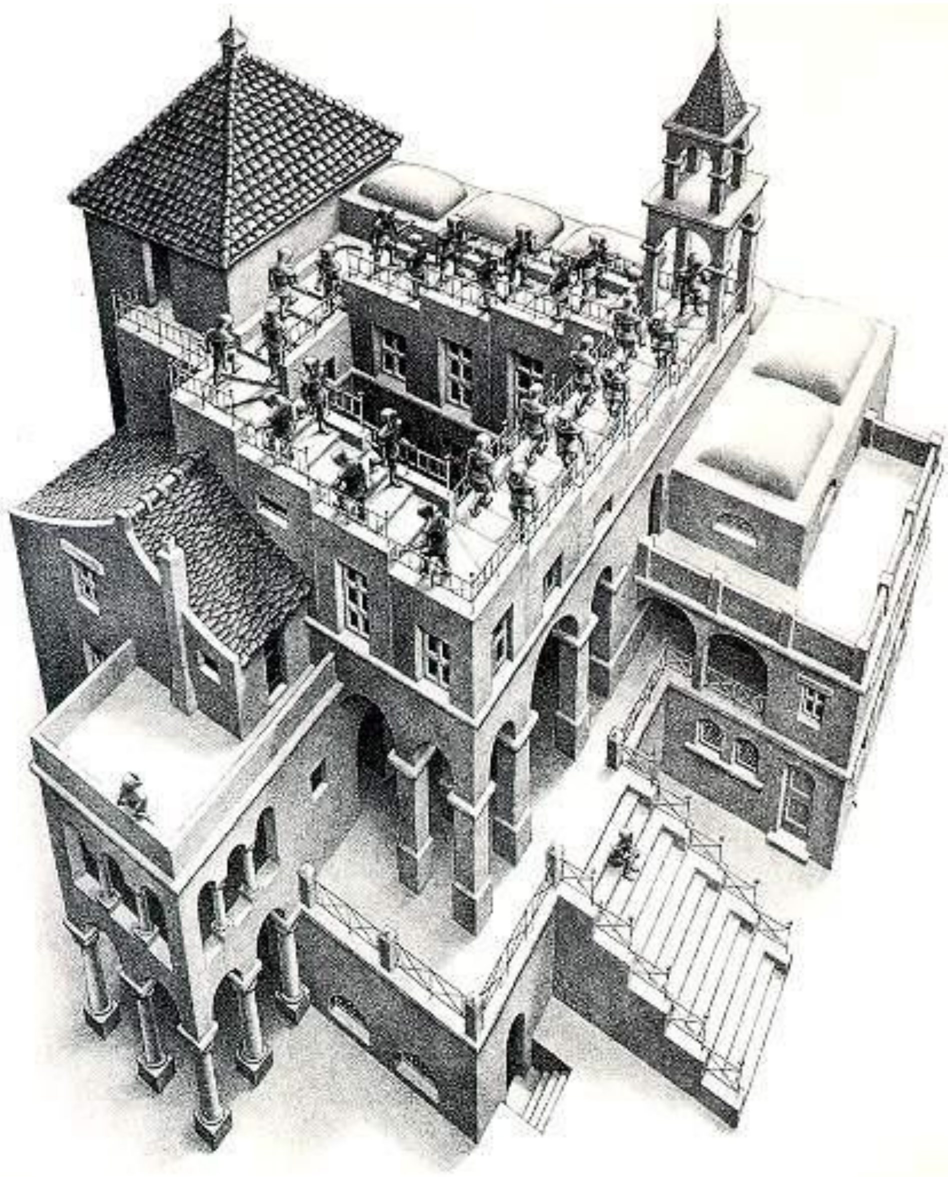


$\bar{B}$ decreasing with time

curly $\bar{E}$-field induced

Both types of E-fields exert forces on charges

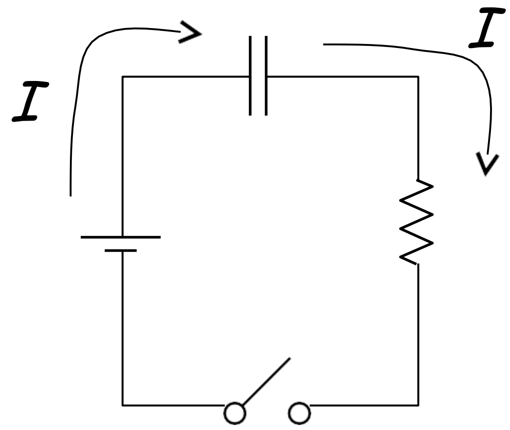
Escher depiction of nonconservative emf



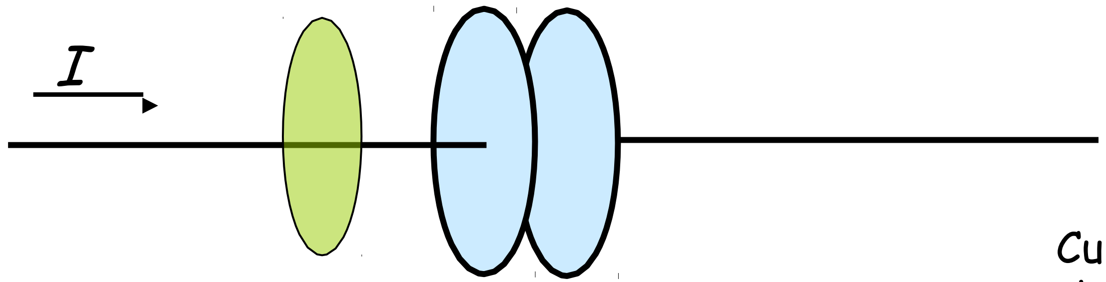
Charging capacitor

When the switch in the circuit below is closed, the capacitor begins charging. While it is charging, is there a current between the plates?

No.



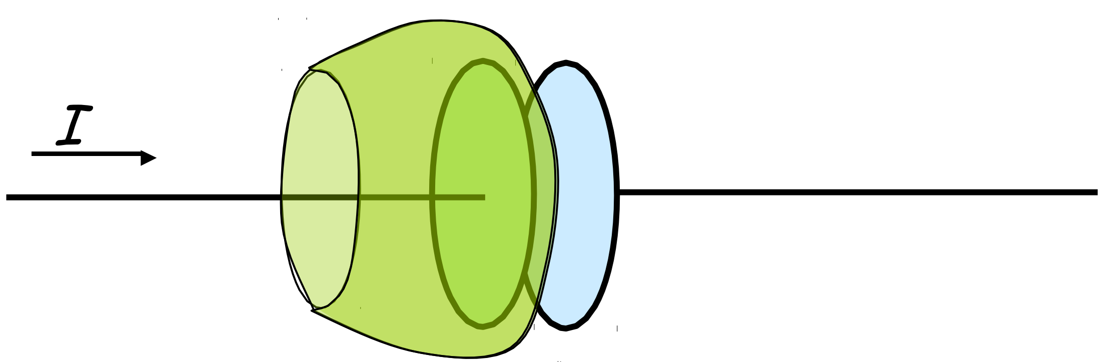
Close up of the capacitor region:



Current intercepting the green area

Ampere's law for the loop shown: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

But what is I decide to use this "bag" as the surface delimited by the loop? What is the current intercepting it??



Displacement current

There is not current, but there is a time-changing electric field between the plates:

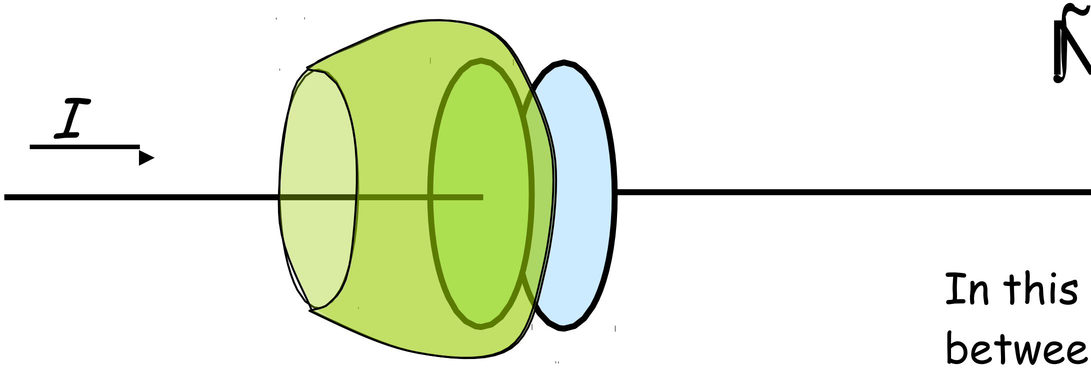
$$E = \frac{q}{\varepsilon_0 A}$$

$$\Rightarrow \quad q = \varepsilon_0 A E = \varepsilon_0 \Phi_E$$

Maxwell proposed to complete Ampere's law with an additional "current": $I_D = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + I_D \right)$$

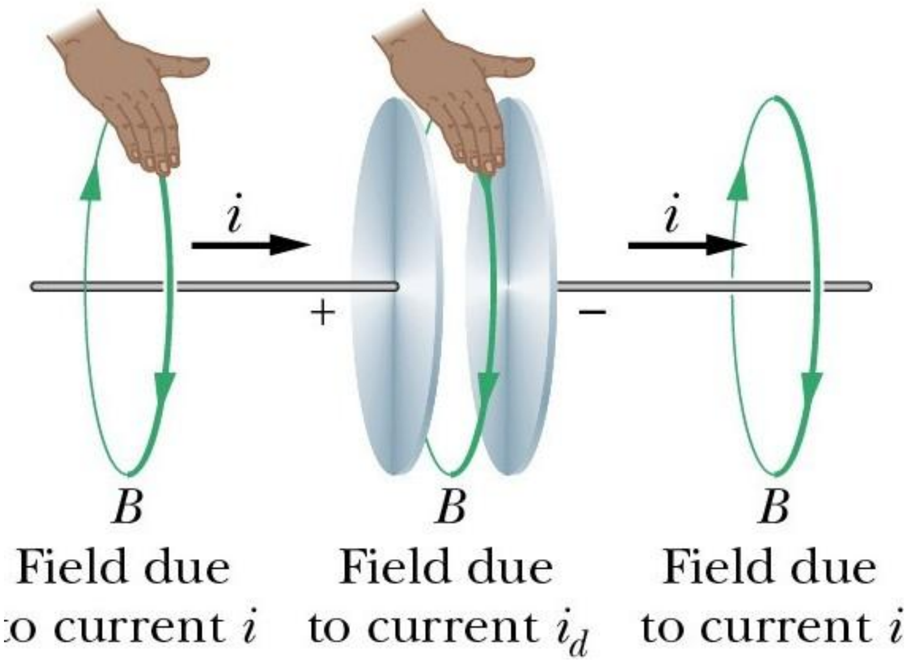


In this case, we have $I_D = I$ between the plates.

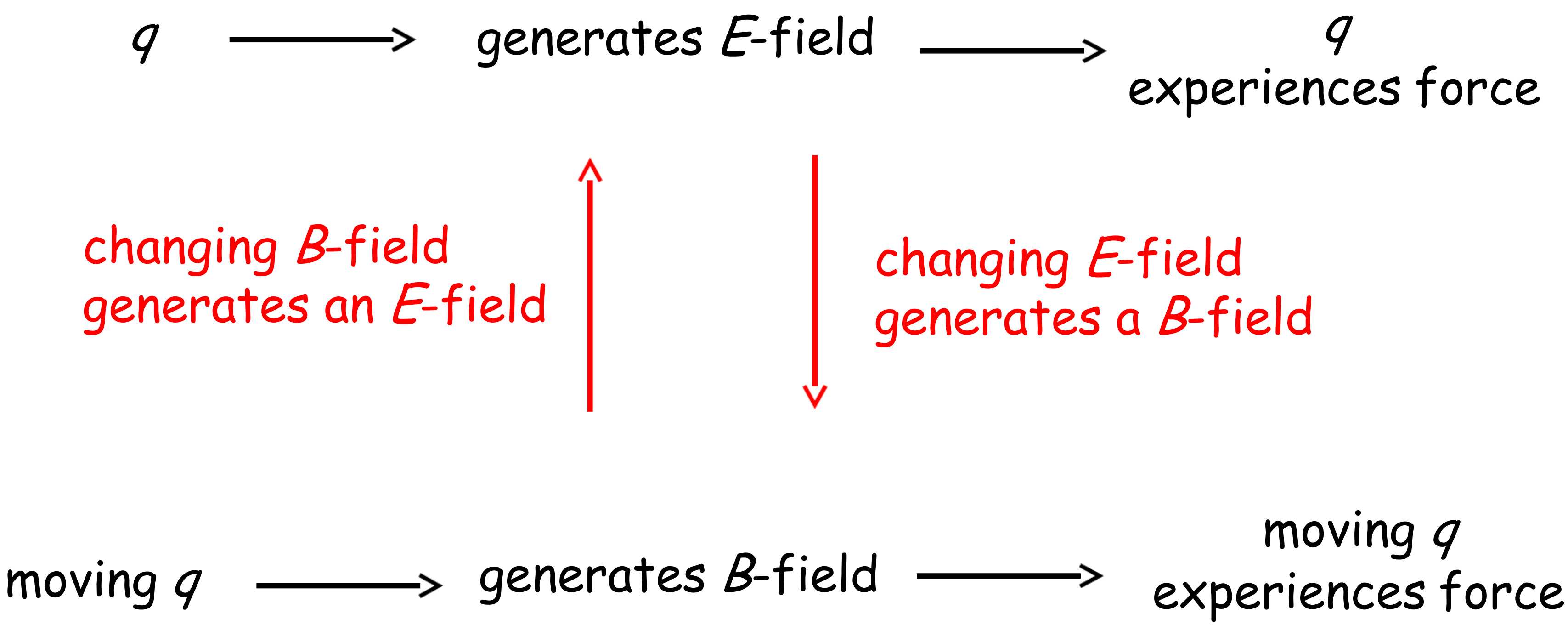
E as a source of B fields

Time-dependent E fields are a source of B fields!

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$



The complete picture



Maxwell's equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Gauss's law for *E*

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's law for *B*

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Ampere's law

In the absence of sources

The symmetry is then very impressive:

$$\oint \vec{E} \cdot d\vec{A} = 0 \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

This is $1/c^2$ (speed of light in vacuum)!!!

There has to be some relation to light here...